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On the scaling behaviour in a logσ–log $\dot{\epsilon}$ diagram

F. POVOLO
Comisión Nacional de Energía Atómica, Dto. de Materiales, Av. del Libertador 8250, (1429) Buenos Aires, Argentina

A. J. MARZOCCA
Comisión de Investigaciones Científicas de la Provincia de Buenos Aires, Argentina

The significance of a scaling behaviour in the creep and stress–relaxation log σ–log $\dot{\epsilon}$ curves is analysed. It is shown that such a property imposes some restrictions on the parameters of the theoretical models. Finally, the formalism is applied to some constitutive equations used in the literature to describe creep and stress–relaxation data.

1. Introduction

Several investigators [1–7] have observed a scaling behaviour in the experimental log σ–log $\dot{\epsilon}$ creep and stress–relaxation curves in various metals and alloys. σ is the applied stress and $\dot{\epsilon}$ the plastic strain rate. This scaling property means that it is possible to superpose by a translation (Δ log σ, Δ log $\dot{\epsilon}$) anyone of the curves onto any of the others, in such a way that the overlapping segments of each curve match within experimental error. Such a scaling behaviour has been taken as a proof of the uniqueness of the log σ–log $\dot{\epsilon}$ curves and of the existence of a plastic equation of state for the material [2]. Povolo and Rubiolo [8] have discussed recently whether the scaling relationship is a sufficient condition to ensure the existence of a state variable, dependent on σ and $\dot{\epsilon}$.

It is the purpose of this paper to analyse the significance of the scaling relationship and the restrictions that such a property imposes on the theoretical models.

2. Theory

Experimentally, a set of log σ–log $\dot{\epsilon}$ curves are obtained, at constant temperature, as a function of a third parameter, which will be represented by γ. This parameter can be, for example, the initial stress for a stress–relaxation experiment, the plastic strain for a creep experiment [7] or the hardness parameter in Hart's phenomenological theory [1, 2] for plastic deformation.

Fig. 1a shows two log σ–log $\dot{\epsilon}$ curves at different γ. If the two curves are related by scaling, then, point A, for example, is translated to point B (or vice versa) along the translation path of slope

$$\mu = \tan\psi = \Delta \log\sigma / \Delta \log\dot{\epsilon} = \text{constant}. \quad (1)$$

μ is the same for all the curves and, for any pair of log σ–log $\dot{\epsilon}$ curves at different γ.

$$\begin{aligned}(\Delta\log\sigma)^2 + (\Delta\log\dot{\epsilon})^2 &= (1+\mu^2)(\Delta\log\dot{\epsilon})^2 \\ &= (1+1/\mu^2)(\Delta\log\sigma)^2 \\ &= F(\Delta\gamma) = \text{constant},\end{aligned} \quad (2)$$

where $F(\Delta\gamma)$ can be obtained experimentally.

The different theoretical expressions, used to describe stress–relaxation or creep behaviour, can be represented in a normalized diagram [9] as

$$f(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta) = 0, \quad (3)$$

where α, $\dot{\epsilon}^*$ and β are parameters that depend on the particular model considered and f is a general function. Equation 3 can be represented, in the normalized plot, as curves parametrized in β. This is shown schematically in Fig. 1b. In Equation 3 β is given as an implicit function of $\dot{\epsilon}/\dot{\epsilon}^*$ and $\alpha\sigma$, to include the cases in which it cannot be obtained explicitly.

The scaling property in the experimental log σ–log $\dot{\epsilon}$ curves, along the translation path given by Equation 1, will impose some restrictions on

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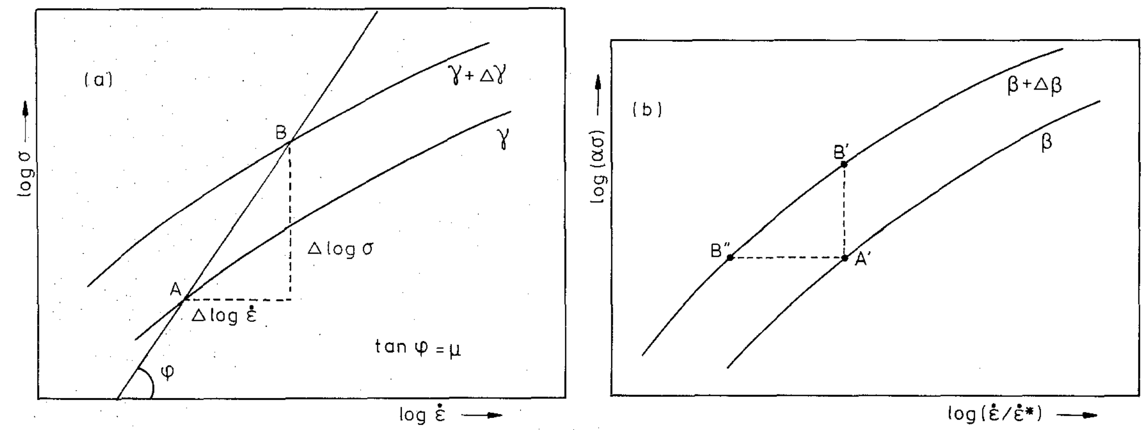

Figure 1 (a) Two experimental log σ–log $\dot{\epsilon}$ curves, at different γ, related by scaling along the translation path of slope μ. (b) Two theoretical curves, at different β, in a normalized plot.

the parameters of the theoretical model. In fact, taking increments of Equation 3 gives

$$\frac{\partial f}{\partial \log(\alpha\sigma)}\Delta \log(\alpha\sigma)+\frac{\partial f}{\partial \log(\dot{\epsilon}/\dot{\epsilon}^*)}\Delta \log(\dot{\epsilon}/\dot{\epsilon}^*)$$

$$+\frac{\partial f}{\partial \log\beta}\Delta\log\beta = 0, \qquad (4)$$

with the additional conditions

$$\frac{\partial f}{\partial \log(\alpha\sigma)}+\frac{\partial f}{\log\beta}\frac{\partial\log\beta}{\partial\log(\alpha\sigma)} = 0, \qquad (5)$$

$$\frac{\partial f}{\partial \log(\dot{\epsilon}/\dot{\epsilon}^*)}+\frac{\partial f}{\partial\log\beta}\frac{\partial\log\beta}{\partial\log(\dot{\epsilon}/\dot{\epsilon}^*)} = 0. \qquad (6)$$

On taking into account Equations 5 and 6, Equation 4 can be written as

$$\frac{\partial\log\beta}{\partial \log(\alpha\sigma)}\Delta \log(\alpha\sigma)+\frac{\partial\log\beta}{\partial \log(\dot{\epsilon}/\dot{\epsilon}^*)}\Delta \log(\dot{\epsilon}/\dot{\epsilon}^*)$$

$$= \Delta\log\beta. \qquad (7)$$

By introducing

$$h = h(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta) = \frac{\partial\log\beta}{\partial\log(\alpha\sigma)}, \qquad (8)$$

$$g = g(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta) = \frac{\partial\log\beta}{\partial\log(\dot{\epsilon}/\dot{\epsilon}^*)}, \qquad (9)$$

Equation 7 can be reduced to

$$h(\Delta\log\alpha+\Delta\log\sigma)+g(\Delta\log\dot{\epsilon}-\Delta\log\dot{\epsilon}^*)$$

$$= \Delta\log\beta. \qquad (10)$$

This equation relates the increments on the experimental variables to the corresponding increments on the theoretical parameters. It must be pointed out that, due to the scaling property expressed by Equations 1 and 2, $\Delta\log\sigma$ and $\Delta\log\dot{\epsilon}$ remain constant, along the curve indicated by γ in Fig. 1a, during the translation.

According to the forms of the functions f and g, Equation 10 leads to different conditions between the increments of $\sigma, \dot{\epsilon}$ and $\alpha, \dot{\epsilon}^*, \beta$:

(a) $h = g \neq k + H(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta)$, i.e. the functions h and g coincide and do not contain an additive constant, k. In this case, Equation 10 can be written as

$$h(\Delta\log\alpha+\Delta\log\sigma+\Delta\log\dot{\epsilon}-\Delta\log\dot{\epsilon}^*)$$

$$= \Delta\log\beta$$

and since $h = h(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta)$ this equation can be satisfied only if

$$\Delta\log\beta = 0 \qquad (11)$$

and

$$\Delta\log\alpha+\Delta\log\sigma+\Delta\log\dot{\epsilon}-\Delta\log\dot{\epsilon}^* = 0. \qquad (12)$$

Equation 11 establishes that β remains constant during the translation and Equation 12 connects the increments of the theoretical parameters α and $\dot{\epsilon}^*$ to the corresponding increments on the experimental variables σ and $\dot{\epsilon}$;

(b) $h = g = k + H(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta)$, where H does not contain an additive constant. Substituting into Equation 10 gives

$$k(\Delta\log\alpha+\Delta\log\sigma+\Delta\log\dot{\epsilon}-\Delta\log\dot{\epsilon}^*)$$

$$+H(\Delta\log\alpha+\Delta\log\sigma+\Delta\log\dot{\epsilon}-\Delta\log\dot{\epsilon}^*)$$

$$= \Delta\log\beta$$

which leads to

$$\Delta\log\beta = 0 \qquad (13)$$

$$\Delta \log \alpha + \Delta \log \sigma + \Delta \log \dot{\epsilon} - \Delta \log \dot{\epsilon}^* = 0, \quad (14)$$

since H is a function of σ and $\dot{\epsilon}$;

(c) $h = g = k$, where k is a constant. Substituting into Equation 10 gives

$$k(\Delta \log \alpha + \Delta \log \sigma + \Delta \log \dot{\epsilon} - \Delta \log \dot{\epsilon}^*) = \Delta \log \beta, \quad (15)$$

(d) $h \neq k_1 + H(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta)$, $g \neq k_2 + G(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta)$ and $h \neq g$; k_1 and k_2 are constants and H and G do not contain additive constants. In this case, Equation 10 can only be satisfied if

$$\Delta \log \beta = 0, \quad (16)$$

$$\Delta \log \alpha + \Delta \log \sigma = 0, \quad (17)$$

$$\Delta \log \dot{\epsilon} - \Delta \log \dot{\epsilon}^* = 0. \quad (18)$$

It is easy to show that the cases

$$h = k_1 + H(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta),$$

$$g \neq k_2 + G(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta), \; h \neq g$$

and

$$h \neq k_1 + H(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta),$$

$$g = k_2 + G(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta), \; h \neq g$$

lead to Equations 16, 17 and 18,

(e) $h = k_1, g \neq k_2 + G(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta)$. Substituting into Equation 10 gives

$$k_1(\Delta \log \alpha + \Delta \log \sigma) + g(\Delta \log \dot{\epsilon} - \Delta \log \dot{\epsilon}^*) = \Delta \log \beta,$$

which can only be satisfied if

$$\Delta \log \dot{\epsilon} - \Delta \log \dot{\epsilon}^* = 0, \quad (19)$$

$$k_1(\Delta \log \alpha + \Delta \log \sigma) = \Delta \log \beta; \quad (20)$$

(f) $h \neq k_1 + H(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta), g = k_2$. Substituting these equations into Equation 10 leads to

$$h(\Delta \log \alpha + \Delta \log \sigma) + k_2(\Delta \log \dot{\epsilon} - \Delta \log \dot{\epsilon}^*) = \Delta \log \beta,$$

which can only be satisfied if

$$\Delta \log \alpha + \Delta \log \sigma = 0, \quad (21)$$

$$k_2(\Delta \log \dot{\epsilon} - \Delta \log \dot{\epsilon}^*) = \Delta \log \beta; \quad (22)$$

(g) $h = k_1$, $g = k_2$. In this case, Equation 10 leads to

$$k_1(\Delta \log \alpha + \Delta \log \sigma) + k_2(\Delta \log \dot{\epsilon} - \Delta \log \dot{\epsilon}^*) = \Delta \log \beta. \quad (23)$$

Equations 17 and 18 allow a determination of the increments of the theoretical parameters $\alpha, \dot{\epsilon}^*$ from the increments of the experimental variables $\sigma, \dot{\epsilon}$. Furthermore, on combining Equations 1, 17 and 18 it is easy to show that

$$\Delta \log \alpha = -\mu \Delta \log \dot{\epsilon}^*$$

or

$$\alpha(\dot{\epsilon}^*)^\mu = \text{constant}, \quad (24)$$

which shows that the scaling condition establishes a relationship between the theoretical parameters α and $\dot{\epsilon}^*$. In addition, combining Equations 1, 2, 17 and 18 leads to

$$F(\Delta\gamma) = (1 + \mu^2)(\Delta \log \dot{\epsilon}^*)^2 = (1 + 1/\mu^2)(\Delta \log \alpha)^2. \quad (25)$$

This equation relates the increments of the experimental parameter γ to the increments of the theoretical parameters. Furthermore, since the translation is performed at constant β, i.e. $\Delta \log \beta = 0$, as the point A translates to B, in Fig. 1a, the point A′, homologous to A, does not move in the normalized plot of Fig. 1b.

For cases (a) and (b) the scaling condition, given by Equations 12 and 14, can be written as

$$(\alpha/\dot{\epsilon}^*) = \text{constant } \dot{\epsilon}^{-(\mu+1)} = \text{constant } \sigma^{-(1+1/\mu)}$$

which shows that the relationship between α and $\dot{\epsilon}^*$ depends on the location of point A, along curve γ of Fig. 1b. This is equivalent to stating that the theoretical expression cannot lead to a scaling behaviour in a log σ–log $\dot{\epsilon}$ diagram.

In the case where $\Delta \log \beta \neq 0$, the increments on the theoretical parameters can be obtained only if the theoretical model provides an additional relationship between the parameters α, $\dot{\epsilon}^*$ and β, i.e. only if two parameters are independent. Then, if $\beta = \beta(\alpha, \dot{\epsilon}^*)$

$$\Delta \log \beta = \frac{\partial \log \beta}{\partial \log \alpha} \Delta \log \alpha + \frac{\partial \log \beta}{\partial \log \dot{\epsilon}^*} \Delta \log \dot{\epsilon}^*$$

which on introducing $a = (\partial \log \beta / \partial \log \alpha)$ and $b = (\partial \log \beta / \partial \log \dot{\epsilon}^*)$ can be written as

$$\Delta \log \beta = a \Delta \log \alpha + b \Delta \log \dot{\epsilon}^*. \quad (26)$$

Case (c) cannot be solved even in this situation since there are only two equations (Equations 15 and 26) and three unknowns ($\Delta \log \alpha$, $\Delta \log \dot{\epsilon}^*$, $\Delta \log \beta$). As for cases (a) and (b), the theoretical model does not give a scaling behaviour in the log σ–log $\dot{\epsilon}$ plot.

A similar situation is found for case (g) which has no solution since

$$f(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta) = \beta - (\alpha\sigma)^{k_1} (\dot{\epsilon}/\dot{\epsilon}^*)^{k_2},$$

and the log σ–log $\dot{\epsilon}$ plot gives parallel straight lines, leading to any translation path.

It can be easily demonstrated that cases (a), (b) and (c), where $h = g$ imply that the function f of Equation 3 must be of the type

$$f = \beta - A[(\alpha\sigma)(\dot{\epsilon}/\dot{\epsilon}^*)] = 0$$

or

$$\beta = A[(\alpha\sigma)(\dot{\epsilon}/\dot{\epsilon}^*)],$$

where A is a function of the product of the variables $\alpha\sigma$ and $\dot{\epsilon}/\dot{\epsilon}^*$, then, in these cases, which also include case (g), either the theoretical expression does not lead to a scaling behaviour in the log σ–log $\dot{\epsilon}$ plot or to any translation path.

For case (e), on combining Equations 1, 2, 19, 20 and 26 it is easy to show that

$$\Delta \log \alpha = \frac{(b - \mu k_1)}{(k_1 - a)} \Delta \log \dot{\epsilon} \qquad (27)$$

$$\Delta \log \dot{\epsilon}^* = \Delta \log \dot{\epsilon} \qquad (28)$$

$$\Delta \log \beta = \frac{k_1(b - a\mu)}{(k_1 - a)} \Delta \log \dot{\epsilon} \qquad (29)$$

$$F(\Delta\gamma) = (1 + \mu^2) \left[\frac{(k_1 - a)}{k_1(b - a\mu)}\right]^2 (\Delta \log \beta)^2. \qquad (30)$$

The last equation relates an increment of the experimental parameter γ to an increment of the theoretical parameter β. Furthermore, since $\Delta \log (\dot{\epsilon}/\dot{\epsilon}^*) = 0$ (Equation 28) as the point A translates to B in Fig. 1a, the point A′ translates to B′, parallel to the $\alpha\sigma$ axis, in the normalized plot of Fig. 1b.

On combining Equations 1, 2, 21, 22 and 26, case (f) leads to

$$\Delta \log \alpha = -\mu \Delta \log \dot{\epsilon} \qquad (31)$$

$$\Delta \log \dot{\epsilon}^* = \frac{(k_2 + \mu a)}{(b + k_2)} \Delta \log \dot{\epsilon} \qquad (32)$$

$$\Delta \log \beta = \frac{k_2(b - \mu a)}{(b + k_2)} \Delta \log \dot{\epsilon} \qquad (33)$$

$$F(\Delta\gamma) = (1 + \mu^2) \left[\frac{(b + k_2)}{k_2(b - \mu a)}\right]^2 (\Delta \log \beta)^2. \qquad (34)$$

The point A′ translates to B″, in the normalized plot of Fig. 1b since $\Delta \log (\alpha\sigma) = 0$.

The principal forms of the function f have been considered and the different scaling conditions are summarized in Table I.

3. Applications

The formalism just described can be applied, for example, to some constitutive equations proposed in the literature for the description of creep and stress–relaxation curves.

Hart *et al.* [2] have proposed the phenomenological constitutive equation

$$\sigma = \sigma^* \exp[-(\dot{\epsilon}/\dot{\epsilon}^*)^{-\lambda}]$$

for the description of plastic behaviour at high homologous temperatures. In this case $\alpha = 1/\sigma^*$ and $\beta = \lambda$ so that

$$h = \frac{\partial \log \beta}{\partial \log (\alpha\sigma)} = \frac{\partial \log \lambda}{\partial \log (\alpha\sigma)} = [\lambda(\dot{\epsilon}/\dot{\epsilon}^*)^{-\lambda} \log(-\dot{\epsilon}/\dot{\epsilon}^*)]^{-1}$$

$$g = \frac{\partial \log \beta}{\partial \log (\dot{\epsilon}/\dot{\epsilon}^*)} = \frac{\partial \log \lambda}{\partial \log (\dot{\epsilon}/\dot{\epsilon}^*)} = \frac{\lambda}{\log [(\dot{\epsilon}/\dot{\epsilon}^*)^{-\lambda}]}.$$

Then, $h \neq g$ and neither of them contain an additive constant so that, according to Table I, the scaling conditions are

$$\Delta \log \lambda = 0$$

$$\Delta \log \sigma^* = \Delta \log \sigma$$

$$\Delta \log \dot{\epsilon}^* = \Delta \log \dot{\epsilon}$$

and

$$\sigma^* = C(\dot{\epsilon}^*)^{\mu}, \qquad (35)$$

where C is a constant. Equation 35 was given, without demonstration, by Hart *et al.* [2].

Johnston and Gilman [10] have proposed the stress–strain rate relationship

$$\dot{\epsilon} = \psi\rho \boldsymbol{b} K(\sigma - \sigma_i)^m \qquad (36)$$

where K and m are material constants at a given temperature; ρ is the mobile dislocation density, $\boldsymbol{b}$ is the Burgers vector, ψ is an orientation factor and σ_i is an internal stress. Equation 36 has been widely used to describe stress–relaxation data and can be written in a normalized form as [9]

$$\alpha\sigma = 1 + (\dot{\epsilon}/\dot{\epsilon}^*)^{\beta} \qquad (37)$$

with

$$\alpha = 1/\sigma_i, \quad \beta = 1/m$$

and

$$\dot{\epsilon}^* = \psi\rho \boldsymbol{b} K(1/\alpha)^{1/\beta}. \qquad (38)$$

TABLE I Relationships to be satisfied by the parameters of the theoretical model, expressed by $f(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta) = 0$, when the $\log \sigma$–$\log \dot{\epsilon}$ experimental curves, parameterized in γ show a scaling behaviour. k_1, k_2 and C are constants; G and H are general functions that do not include an additive constant and A is a function of the product $(\alpha\sigma)(\dot{\epsilon}/\dot{\epsilon}^*)$. $a = \partial \log \beta / \partial \log \alpha$ and $b = \partial \log \beta/\partial \log \dot{\epsilon}^*$ in the additional relationship $\beta = \beta(\alpha, \dot{\epsilon}^*)$ provided by the theoretical model

$f(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta) = 0$	$h = \dfrac{\partial \log \beta}{\partial \log (\alpha\sigma)}$	$g = \dfrac{\partial \log \beta}{\partial \log (\dot{\epsilon}/\dot{\epsilon}^*)}$	Scaling conditions
$\beta = A[(\alpha\sigma)(\dot{\epsilon}/\dot{\epsilon}^*)]$	$h = g$	$h = g$	No scaling or any translation path
$\beta = C(\alpha\sigma)^{k_1}(\dot{\epsilon}/\dot{\epsilon}^*)^{k_2}$	k_1	k_2	
$\beta \neq A[(\alpha\sigma)(\dot{\epsilon}/\dot{\epsilon}^*)]$	$\neq k_1 + H\left(\alpha\sigma, \dfrac{\dot{\epsilon}}{\dot{\epsilon}^*}, \beta\right)$	$\neq k_2 + G\left(\alpha\sigma, \dfrac{\dot{\epsilon}}{\dot{\epsilon}^*}, \beta\right)$	$\Delta \log \beta = 0$ $\Delta \log \alpha = -\Delta \log \sigma = -\mu\Delta \log \dot{\epsilon}$ $\Delta \log \dot{\epsilon}^* = \Delta \log \dot{\epsilon}$
	$= k_1 + H$	$\neq k_2 + G$	$\alpha(\dot{\epsilon}^*)^{\mu} = \text{constant}$
	$\neq k_1 + H$	$= k_2 + G$	$F(\Delta\gamma) = (1 + \mu^2)(\Delta \log \dot{\epsilon}^*)^2$ $= (1 + 1/\mu^2)(\Delta \log \alpha)^2$
	k_1	$\neq k_2 + G$	$\Delta \log \alpha = \dfrac{(b - \mu k_1)}{(k_1 - a)} \Delta \log \dot{\epsilon}$ $\Delta \log \dot{\epsilon}^* = \Delta \log \dot{\epsilon}$ $\Delta \log \beta = \dfrac{k_1(b - \mu a)}{(k_1 - a)} \Delta \log \dot{\epsilon}$ $F(\Delta\gamma) = (1 + \mu^2)\left[\dfrac{(k_1 - a)}{k_1(b - \mu a)}\right]^2 (\Delta \log \beta)^2$
	$\neq k_1 + H$	k_2	$\Delta \log \alpha = -\mu\Delta \log \dot{\epsilon}$ $\Delta \log \dot{\epsilon}^* = \dfrac{(k_2 + \mu a)}{(b + k_2)} \Delta \log \dot{\epsilon}$ $\Delta \log \beta = \dfrac{k_2(b - \mu a)}{(b + k_2)} \Delta \log \dot{\epsilon}$ $F(\Delta\gamma) = (1 + \mu^2)\left[\dfrac{(b + k_2)}{k_2(b - \mu a)}\right]^2 (\Delta \log \beta)^2$

From Equation 37 it is easily seen that

$$h = \frac{\partial \log \beta}{\partial \log (\alpha\sigma)} = \frac{\alpha\sigma}{(\dot{\epsilon}/\dot{\epsilon}^*)\beta \ln (\dot{\epsilon}/\dot{\epsilon}^*)}$$

$$g = \frac{\partial \log \beta}{\partial \log (\dot{\epsilon}/\dot{\epsilon}^*)} = -\frac{\beta}{\ln (\dot{\epsilon}/\dot{\epsilon}^*)},$$

then, $h \neq g$ and neither of them contain an additive constant. From Table I

$$\Delta \log \beta = -\Delta \log m = 0$$

$$\Delta \log \sigma_i = \Delta \log \sigma$$

$$\Delta \log \dot{\epsilon}^* = \Delta \log \dot{\epsilon}\,.$$

In addition, from the relationship $\beta = \beta(\alpha, \dot{\epsilon}^*)$, given by Equation 38, it is easily seen that

$$\mu = 1/m,$$

i.e. if Equation 37 (or 36) leads to a scaling behaviour in a $\log \sigma$–$\log \dot{\epsilon}$ plot, $1/m$ should give the slope of the translation path if the dislocation density, ρ, remains constant.

Finally, Friedel [11] has developed a creep theory based on the thermally activated glide of dislocation loops cutting trees of the dislocation forest. When no elastic interaction between the trees and the moving loop is present, the strain rate is given by

$$\dot{\epsilon} = \rho v (2\alpha' G/lb)^{-2/3} (\sigma - \sigma_i)^{2/3} \times \exp \left[\frac{-2U_j + bd(2\alpha' G b l^2)^{1/3} (\sigma - \sigma_i)^{2/3}}{kT}\right] \tag{39}$$

where U_j is the energy to form the two jogs during the cutting process, d is the slipping distance necessary for the jogs to be formed, G is shear modulus, ρ is the density of moving loops, v is Debye frequency, $\boldsymbol{b}$ is the Burgers vector, l is the average spacing between dislocations and α' is a geometrical factor. σ_i is a long-range frictional stress due to elastic stresses and is given by

$$\sigma_i = Gb/\beta' l, \tag{40}$$

where β' is a geometrical constant.

Equation 39 can be normalized to

$$\dot{\epsilon}/\dot{\epsilon}^* = (\alpha\sigma - 1)^{2/3} \exp [\beta(\alpha\sigma - 1)^{2/3}] \tag{41}$$

where

$$\dot{\epsilon}^* = \rho v(2\alpha' G/bl)^{-2/3} \sigma_i^{2/3} \exp(-2U_j/kT) \tag{42}$$

$$\beta = (bd/kT)(2\alpha' Gbl^2)^{1/3} \sigma_i^{2/3} \tag{43}$$

$$\alpha = 1/\sigma_i = \beta' l/Gb. \tag{44}$$

From Equations 41 to 44 it is seen that

$$\beta = C\dot{\epsilon}^* \tag{45}$$

with

$$C = (2\alpha' Gb^2 d/\rho vkT) \exp(2U_j/kT).$$

On differentiating Equation (41) it is easy to show that

$$h = \frac{\partial \log \beta}{\partial \log (\alpha\sigma)} = -(2/3) \frac{\alpha\sigma}{(\alpha\sigma - 1)} \times \left\{1 + \frac{1}{\ln [\dot{\epsilon}/\dot{\epsilon}^*(\alpha\sigma - 1)^{2/3}]}\right\} \tag{46}$$

$$g = \frac{\partial \log \beta}{\partial \log (\dot{\epsilon}/\dot{\epsilon}^*)} = \frac{1}{\ln [\dot{\epsilon}/\dot{\epsilon}^*(\alpha\sigma - 1)^{2/3}]}. \tag{47}$$

Then, $h \neq g$ and neither h nor g contains an additive constant. From Table I it is seen that $\Delta \log \beta = 0$ that implies, according to Equations 45 and 18, $\Delta \log \dot{\epsilon} = \Delta \log \dot{\epsilon}^* = 0$. Equation 39 leads to a scaling behaviour with a translation path parallel to the σ axis, i.e. $\mu = \infty$.

4. Discussion and conclusions

Owing to the scaling property, the individual $\log \sigma$–$\log \dot{\epsilon}$ curves, at different γ, can be superposed by translations along the translation path of slope μ, leading to a master curve corresponding to a given γ [2]. Such a master curve extends the experimental range and can be used to obtain the function $f(\alpha\sigma, \dot{\epsilon}/\dot{\epsilon}^*, \beta) = 0$ of the theoretical model. Once the parameters α, $\dot{\epsilon}^*$ and β, corresponding to a given γ, are known the parameters for the rest of the individual curves can be obtained, from $\Delta \log \dot{\epsilon}$ or $\Delta \log \sigma$, by using the scaling conditions given in Table I. This procedure can be used also if, by some limiting procedure, the parameters for one of the individual curves can be found. This method was used by Povolo and Marzocca [12] to obtain the theoretical parameters for $\log \sigma$–$\log \dot{\epsilon}$ creep curves measured in Zircaloy-4.

From the theoretical point of view, a scaling behaviour in the $\log \sigma$–$\log \dot{\epsilon}$ curves imposes restrictions on the parameters of the theoretical model. If the data, for example, are described by Hart's model and show a scaling behaviour, the parameter λ must be the same for all the individual curves and the parameter σ^* must be related to $\dot{\epsilon}^*$, according to Equation 35. If, on the other hand, the data are described by Friedel's theory (Equation 39), the translation path must be parallel to the σ axis.

Finally, it should be pointed out that the cases considered are not exhaustive and are given as a guide. In fact, more sophisticated relationships between h and g might occur as, for example,

$$h = k_1 + k_2 g,$$

which substituted into Equation 10 and taking into account Equations 1 and 26 leads to

$$\Delta \log \alpha = \frac{(k_1\mu - \mu k_2 b - b)}{(k_2 b - k_1 + a)} \Delta \log \dot{\epsilon}$$

$$\Delta \log \dot{\epsilon}^* = \frac{(k_2\mu - k_1 + a)}{(k_2 b - k_1 + a)} \Delta \log \dot{\epsilon}$$

$$\Delta \log \beta = \frac{k_1(k_1\mu - b - k_1^2\mu + a\mu)}{(k_2 b - k_1 + a)} \Delta \log \dot{\epsilon}.$$

Any relationship between h and g, not considered in Table I, can be analysed following the procedure described in the paper.

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